

Maximal Acceleration Effects in Reissner-Nordström Space.

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Abstract

The dynamics of a relativistic particle in a Reissner-Nordström background is studied using Caianiello model with maximal acceleration. The behaviour of the particle, embedded in a new effective geometry, changes with respect to the classical scenario because of the formation of repulsive potential barriers near the horizon. Black hole formation by accretion of massive particles is not therefore a viable process in the model. At the same time, the naked singularity remains largely unaffected by maximal acceleration corrections.

PACS: 04.50.+h, 04.70.Bw

Keywords: Reissner-Nordström metric, Quantum Geometry, Maximal Acceleration

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A model developed by Caianiello and collaborators [1] provides quantum mechanics with a geometrical framework and delves into a number of fundamental issues such as the unification of general relativity and quantum mechanics and the regularization of field equations. In particular, the model interprets quantization as curvature of the eight-dimensional space-time tangent bundle TM, incorporates the Born reciprocity principle and the notion that the proper acceleration of massive particles has an upper limit \mathcal{A}_m .

Classical and quantum arguments supporting the existence of a maximal acceleration (MA) have long been discussed in the literature [2]. MA also appears in the context of Weyl space [3] and of a geometrical analogue of Vigier's stochastic theory [4]. Its existence would rid black hole entropy of ultraviolet divergencies [5],[6], and circumvent inconsistencies associated with the application of the point-like concept to relativistic quantum particles [7].

Some authors regard \mathcal{A}_m as a universal constant fixed by Planck's mass [8],[9], but a direct application of Heisenberg's uncertainty relations [10],[11] as well as the geometrical interpretation of the quantum commutation relations given by Caianiello, suggest that \mathcal{A}_m be fixed by the rest mass of the particle itself according to $\mathcal{A}_m = 2mc^3/\hbar$.

A limit on the acceleration also occurs in string theory. Here the upper limit manifests itself through Jeans-like instabilities [12] which occur when the acceleration induced by the background gravitational field is larger than a critical value $a_c = (m\alpha)^{-1}$ for which the string extremities become causally disconnected [13]. m is the string mass and α is the string tension. Frolov and Sanchez [14] have then found that a universal critical acceleration $a_c = (m\alpha)^{-1}$ must be a general property of strings.

While in all these instances the critical acceleration results from the interplay of the Rindler horizon with the finite extension of the particle [15],[16], in the Caianiello model MA is a basic physical property of all massive particles which appears from the out-set in the physical laws. At the same time the model introduces an invariant interval in TM that leads to a regularization of the field equations that does not require a fundamental length as in [17] and does therefore preserve the continuum structure of space-time.

Applications of Caianiello's model include cosmology [9], where the initial singularity can be avoided while preserving inflation, the dynamics of accelerated strings [18], the energy spectrum of a uniformly accelerated particle [15], the periodic structure as a function of momentum in neutrino oscillations [19] and the expansion of the very early universe [13]. The model also makes the

metric observer-dependent, as conjectured by Gibbons and Hawking [20].

The extreme large value that \mathcal{A}_m takes for all known particles makes a direct test of the model very difficult. Nonetheless a direct test that uses photons in a cavity has also been suggested [21]. More recently, we have worked out the consequences of the model for the classical electrodynamics of a particle [22], the mass of the Higgs boson [23] and the Lamb shift in hydrogenic atoms [24]. In the last instance the agreement between experimental data and MA corrections is very good for H and D . For He^+ the agreement between theory and experiment is improved by 50% when MA corrections are included. MA effects in muonic atoms also appear to be measurable [25].

In all these works space-time is endowed with a causal structure obtained by means of an embedding procedure pioneered in [19] and discussed at length in [15],[26]. The procedure stipulates that the line element experienced by an accelerating particle, in the presence of gravity, is given by

$$d\tau^2 = \left(1 + \frac{g_{\mu\nu}\ddot{x}^\mu\ddot{x}^\nu}{\mathcal{A}_m^2}\right) g_{\alpha\beta}dx^\alpha dx^\beta \equiv \sigma^2(x)g_{\alpha\beta}dx^\alpha dx^\beta, \quad (1)$$

where $\ddot{x}^\mu = d^2x^\mu/ds^2$ is the, in general, non-covariant acceleration of a particle along its worldline. As a consequence, the effective space-time geometry experienced by accelerated particles exhibits mass-dependent corrections, which in general induce curvature, give rise to a mass-dependent violation of the equivalence principle and vanish in the classical limit $(\mathcal{A}_m)^{-1} = \frac{\hbar}{2mc^3} \rightarrow 0$.

We have recently studied the modifications produced by MA in the motion of a test particle in a Schwarzschild field [26] and found that these account for the presence of a spherical shell, external to the Schwarzschild sphere, that is forbidden to any classical particle and hampers the formation of a black hole. The analogous occurrence of a classically impenetrable shell was derived by Gasperini as a consequence of the breaking of the local $SO(3,1)$ symmetry [27]. The shell remains impervious to quantum, scalar particles [28].

Before proceeding, a few comments are in order [26]. The effective theory presented is intrinsically non-covariant, as is the four-acceleration that appears in $\sigma^2(x)$. In addition $\sigma^2(x)$ could be eliminated from (1) by means of a coordinate transformation if one applied the principles of general relativity to this effective theory. On the contrary, the embedding procedure requires that $\sigma^2(x)$ be present in (1) and that it be calculated in the same coordinates

of the unperturbed gravitational background. Nonetheless the choice of \ddot{x}^μ is supported by the derivation of \mathcal{A}_m from quantum mechanics, by special relativity and by the weak field approximation to general relativity. The model is not intended, therefore, to supersede general relativity, but rather to provide a way to calculate the quantum corrections to the structure of space-time implied by (1). Remarkably, the results of [26] persist in isotropic coordinates [28].

For convenience, the natural units $\hbar = c = G = 1$ are used below.

The purpose of this paper is to extend the calculation of the MA corrections to the Reissner–Nordström metric. The problem is not trivial. This metric, of form

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r} + \frac{e^2}{r^2}} - r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (2)$$

does, in fact, contain two lengths, the mass of the central object M and its charge e . The global space structure therefore depends on the ratio of these two lengths. In addition, the solution does not pertain to vacuum and contains a naked singularity for certain values of the parameters.

Apart from the trivial case of vanishing e , which yields the Schwarzschild metric, three cases must be distinguished. If $|e| < M$, the Schwarzschild horizon moves from $2M$ to $r_A = M + \sqrt{M^2 - e^2}$, while a new horizon forms at the origin. Its position is $r_B = M - \sqrt{M^2 - e^2}$. In the region between these two horizons, the component g_{00} of the metric tensor is negative and particles can only move towards the origin. Elsewhere, particles are allowed to move away from the origin.

When $|e| = M$, the two horizons merge at $r = M$. Yet particles at this point cannot leave and the region inside the sphere of radius M is still not accessible to an external observer.

Finally, for charges larger than the critical value, no horizon exists, the metric (2) can be used in the whole space and particles can move in and out without restrictions. The solution then corresponds to a naked singularity.

In order to calculate the corrections to the Reissner–Nordström field experienced by a particle initially at infinity and falling toward the origin along a geodesic, one must calculate the metric induced by the embedding procedure discussed in [26] to first order in the parameter \mathcal{A}_m^{-2} . On choosing $\theta = \pi/2$,

one finds

$$\sigma^2(r) = 1 + \frac{1}{\mathcal{A}_m^2} \left[\left(1 - \frac{2M}{r} + \frac{e^2}{r^2} \right) \dot{t}^2 - \left(1 - \frac{2M}{r} + \frac{e^2}{r^2} \right)^{-1} \dot{r}^2 - r^2 \dot{\varphi}^2 \right]. \quad (3)$$

The components of the four-acceleration can be easily obtained by following the same steps as in the Schwarzschild case [26]. When these are substituted into (3), the conformal factor becomes

$$\begin{aligned} \sigma^2(r) = 1 + \frac{1}{\mathcal{A}_m^2} & \left\{ - \frac{\left(-\frac{M}{r^2} + \frac{e^2}{r^3} + \frac{\tilde{L}^2}{r^3} - \frac{3ML^2}{r^4} + \frac{2e^2\tilde{L}^2}{r^5} \right)}{\left(1 - \frac{2M}{r} + \frac{e^2}{r^2} \right)} + \right. \\ & \left. + \left[\tilde{E}^2 \frac{\left(\frac{2M}{r^2} - \frac{2e^2}{r^3} \right)^2}{\left(1 - \frac{2M}{r} + \frac{e^2}{r^2} \right)^3} - \frac{4\tilde{L}^2}{r^4} \right] \left[\tilde{E}^2 - \left(1 - \frac{2M}{r} + \frac{e^2}{r^2} \right) \left(1 + \frac{\tilde{L}^2}{r^2} \right) \right] \right\}. \quad (4) \end{aligned}$$

As in [26], \tilde{E} and \tilde{L} are the energy and the angular momentum per unit of test particle mass.

Bearing in mind that the magnitude of the energy-momentum four-vector is given by the rest mass m of the particle according to $g_{\alpha\beta}p^\alpha p^\beta = m^2$ and using the modified metric (1), one can introduce an effective potential V_{eff} by means of the equation

$$\left(\frac{dr}{d\tau} \right)^2 = \tilde{E}^2 - V_{eff}^2, \quad (5)$$

where

$$V_{eff}^2 = \tilde{E}^2 - \frac{\tilde{E}^2}{\sigma^4(r)} + \frac{1}{\sigma^2(r)} \left(1 - \frac{2M}{r} + \frac{e^2}{r^2} \right) \left(1 + \frac{\tilde{L}^2}{r^2\sigma^2(r)} \right). \quad (6)$$

In the limit $\mathcal{A}_m \rightarrow \infty$, or equivalently $\sigma^2(r) \rightarrow 1$, we recover the classical Reissner-Nordström potential. Moreover, as $e \rightarrow 0$, (6) reduces to the effective potential of [26].

In the following we introduce the adimensional variable $\rho = r/M$ and the parameters $\epsilon = (M\mathcal{A}_m)^{-1}$, $\lambda = \tilde{L}/M$ and $\eta = e/M$. Notice that $V_{eff}^2(\rho) \rightarrow 1$ as $\rho \rightarrow \infty$ and $V_{eff}^2(\rho) \rightarrow \tilde{E}^2$ as $\rho \rightarrow 0$. The main features of (6) can be derived and compared with the corresponding corrections determined for the Schwarzschild case.

a) $|\eta| < 1$ and $\lambda = 0$. Since the component $g_{00} = \left(1 - \frac{2}{\rho} + \frac{\eta^2}{\rho^2}\right)$ of the metric tensor vanishes at $\rho_A = 1 + \sqrt{1 - \eta^2}$ and $\rho_B = 1 - \sqrt{1 - \eta^2}$, it follows from (4) that the conformal factor diverges as g_{00}^{-3} for the same values of ρ . $\sigma^2(\rho)$ always appears in the denominators of the effective potential which thus equals \tilde{E}^2 .

Fig. 1 shows $V_{eff}^2(\rho)$ for a radially incoming particle ($\lambda = 0$, $\eta^2 = 0.1$) and different values of the energy.

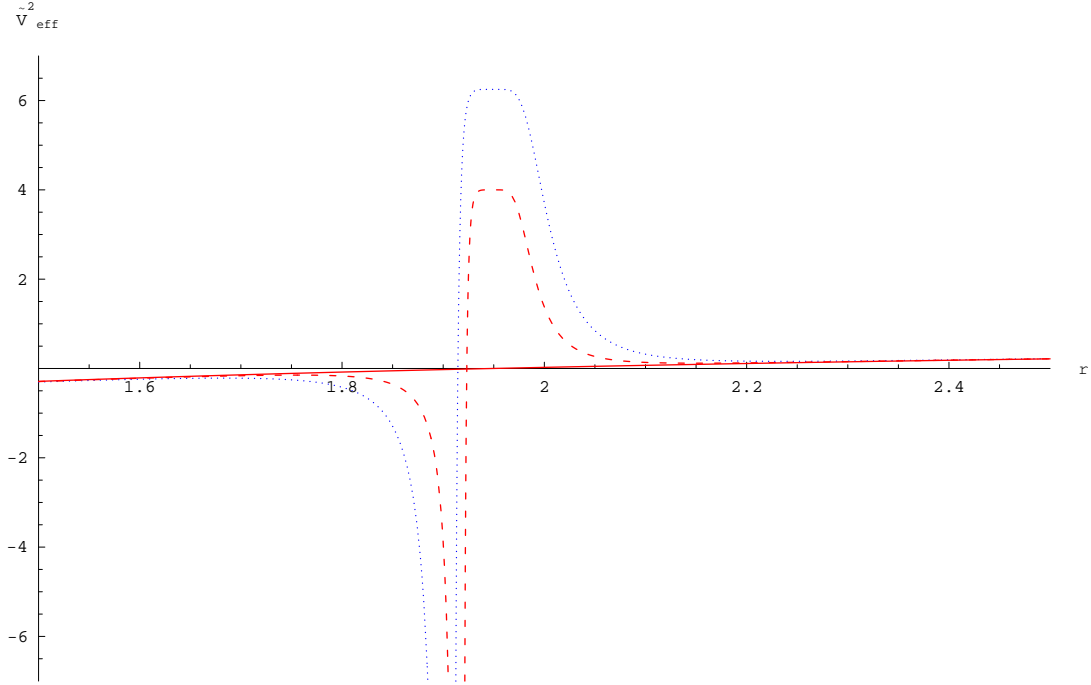


Figure 1: $V_{eff}^2(\rho)$ per unit of particle rest mass m for $\lambda = 0$, $\epsilon = 10^{-3}$, $\eta^2 = 0.1$, $\tilde{E} = 2.5$ (dotted line) and $\tilde{E} = 2$ (dashed line). The solid line refers to the usual Reissner Nordström potential.

As in the Schwarzschild case [26], a shell is formed in ρ_A that prevents external particles from falling into the no-return region. The shell is dynamical in origin, is impenetrable to classical particles and remains so at higher orders of approximation in \mathcal{A}_m^{-2} .

An expansion of (6) in the neighborhood of $\rho_A = 1 + \sqrt{1 - \eta^2}$ shows that the potential barrier behaves like

$$V_{eff}^2(\rho) = \tilde{E}^2 + \frac{4f(\eta)}{\tilde{E}^4 \epsilon^2} (\rho - \rho_A)^4 + O((\rho - \rho_A)^5), \quad (7)$$

where

$$f(\eta) = \frac{(1 - \eta^2) (1 + \sqrt{1 - \eta^2})^5}{8(32 - 72\eta^2 + 54\eta^4 - 15\eta^6) + 9\eta^8 + \sqrt{1 - \eta^2} (4 - \eta^2) (64 - 96\eta^2 + 36\eta^4 - \eta^6)}.$$

$V_{eff}^2(\rho)$ clearly has the minimum \tilde{E}^2 on the horizon ρ_A . On the left of the barrier, there is a divergence originated by a zero in $\sigma^2(\rho)$. This divergence is always negative because the dominant term in (6), $-\frac{\tilde{E}^2}{\sigma^4(\rho)}$, is negative for the corresponding values of ρ . To the left of the divergence, the effective potential returns to values very close to those of the Reissner-Nordström potential. The shape of V_{eff}^2 is rather insensitive to variations of the parameters. In particular, the barrier and the following divergence never change.

Near ρ_B the situation is more complex since the conformal factor has two, or four more roots (depending on the values of η and \tilde{E}), one on the left of ρ_B and one, or three, on the right. The detailed behaviour of the corresponding divergences is, however, irrelevant in the present context because no classical particle can reach the region to the left of the potential barrier near ρ_A .

b) $|\eta| < 1$ and $\lambda \neq 0$. As in a) above, the effective potential takes the value \tilde{E}^2 when ρ tends to zero and to the horizons ρ_A and ρ_B . On the right of ρ_A the potential barrier is finite when the angular momentum λ is lower than a critical value $\lambda_c(\epsilon, \tilde{E}, \eta)$, but infinite if $\lambda > \lambda_c$. This barrier prevents external particles from falling into the no-return region. Fig. 2 shows the behaviour of $V_{eff}^2(\rho)$ in the neighborhood of the horizon ρ_A for two different values of λ , $\lambda_1 > \lambda_c$ and $\lambda_2 < \lambda_c$.

The singularity structure on the left of ρ_A is as complex as in case a), and likewise irrelevant.

c) $|\eta| = 1$. At this critical value, g_{00} has a double root at $\rho = 1$. Here, the effective potential forms the usual barrier as $\sigma^2(\rho)$ diverges. This is shown in Fig. 3 (for $\lambda = 0$) and in Fig. 4 (for $\lambda \neq 0$). An expansion of (6) in the

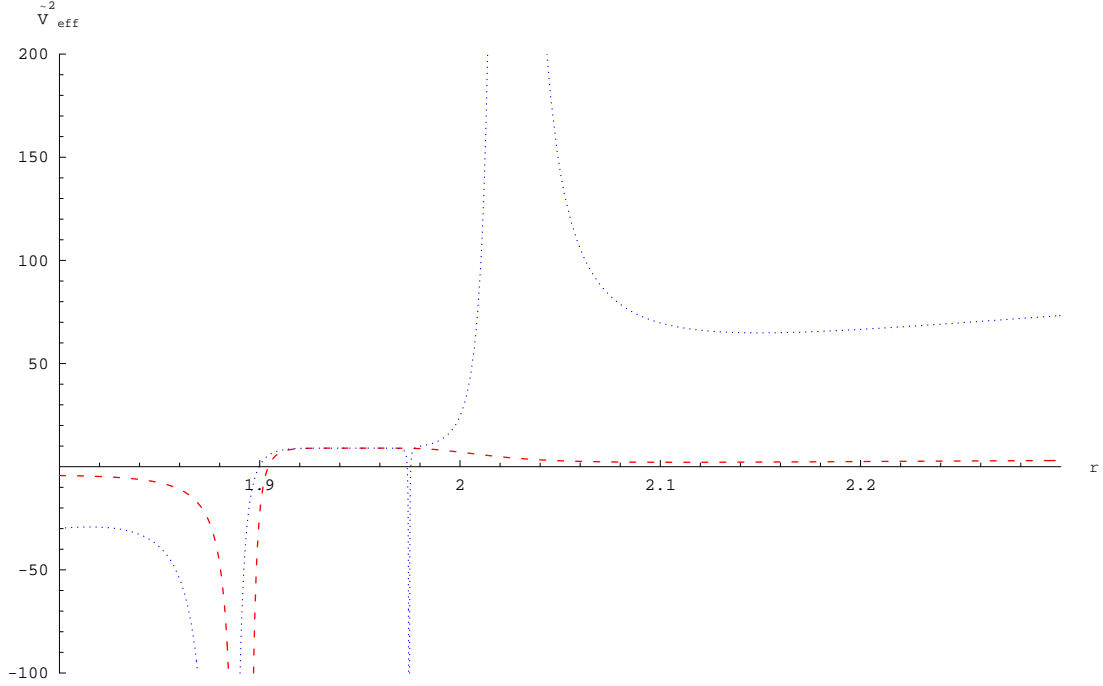


Figure 2: $V_{eff}^2(\rho)$ per unit of particle rest mass m for $\tilde{E} = 3$, $\epsilon = 10^{-3}$, $\eta^2 = 0.1$, $\lambda_1 = 50$ (dotted line) and $\lambda_2 = 10$ (dashed line).

neighborhood of $\rho_A = 1$ yields the potential barrier

$$V_{eff}^2(\rho) = \tilde{E}^2 + \frac{4}{\tilde{E}^4 \epsilon^2} (\rho - \rho_A)^6 + O((\rho - \rho_A)^7)$$

which, clearly, has the minimum \tilde{E}^2 on the horizon $\rho_A = 1$. Near the origin there still is the singularity produced by the zero of the conformal factor. Here too the sign is determined by the values of \tilde{E} and λ ; it becomes positive when $\lambda^2 > \frac{\tilde{E}^2 \rho_0^2}{(1 - \rho_0)^2}$, where ρ_0 is the point at which $\sigma^2(\rho)$ vanishes.

d) $|\eta| > 1$. g_{00} is positive for all values of ρ . There are no more horizons and the barrier shrinks (Fig. 5) until it disappears. Although the naked singularity is no longer point-like, its behaviour remains unaffected by MA corrections.

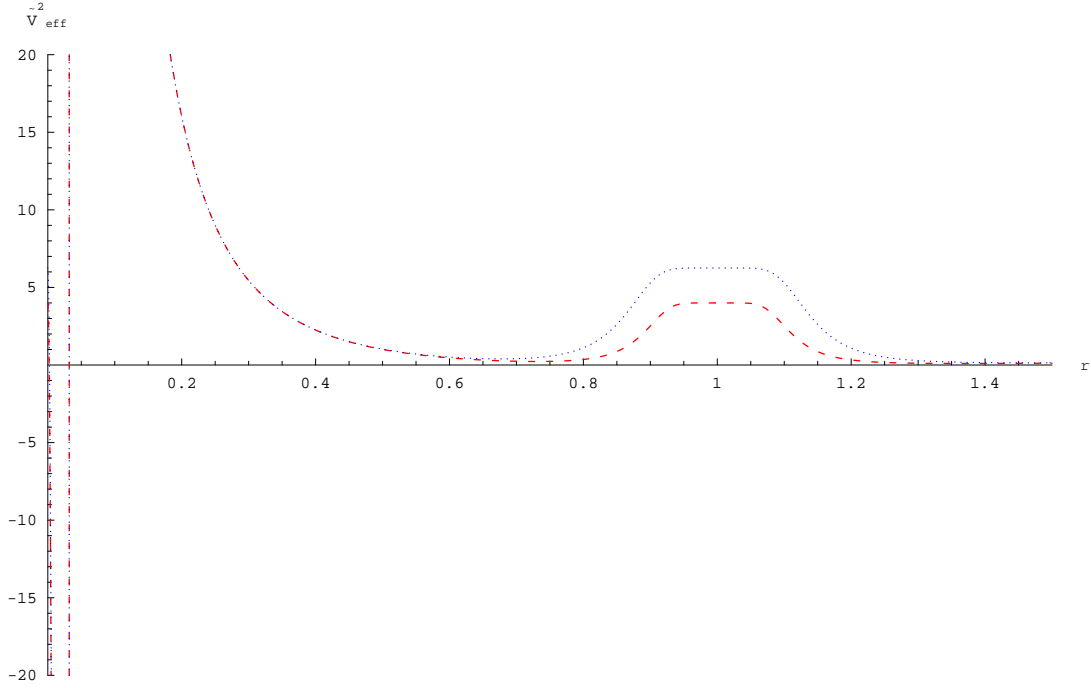


Figure 3: $V_{eff}^2(\rho)$ per unit of particle rest mass m for $\lambda = 0$, $\epsilon = 10^{-3}$, $\eta^2 = 1$, $\tilde{E} = 2.5$ (dotted line) and $\tilde{E} = 2$ (dashed line). Note that there is only one singular point at $\rho \simeq 0.003$, where $V_{eff}^2 \rightarrow -\infty$

For a black hole with 3.3 solar masses, the critical value for the central charge is 5.7×10^{20} coulomb. Obviously, this value can never be reached in the normal case irrespective of the relative signs of the charges of m and M . If, in fact, matter and black hole had charges of the same sign, then the electromagnetic repulsion would prevent the fall of matter. If the signs were opposite, then particles would be strongly attracted and would tend to neutralize the whole system. For this reason real black holes can only be “low charged” and should be described by the $|e| < M$ case.

The corrections to the Reissner–Nordström metric induced by the presence of MA do indeed represent an interesting confirmation of the physics already discussed for the Schwarzschild case. Apart from the classical shift

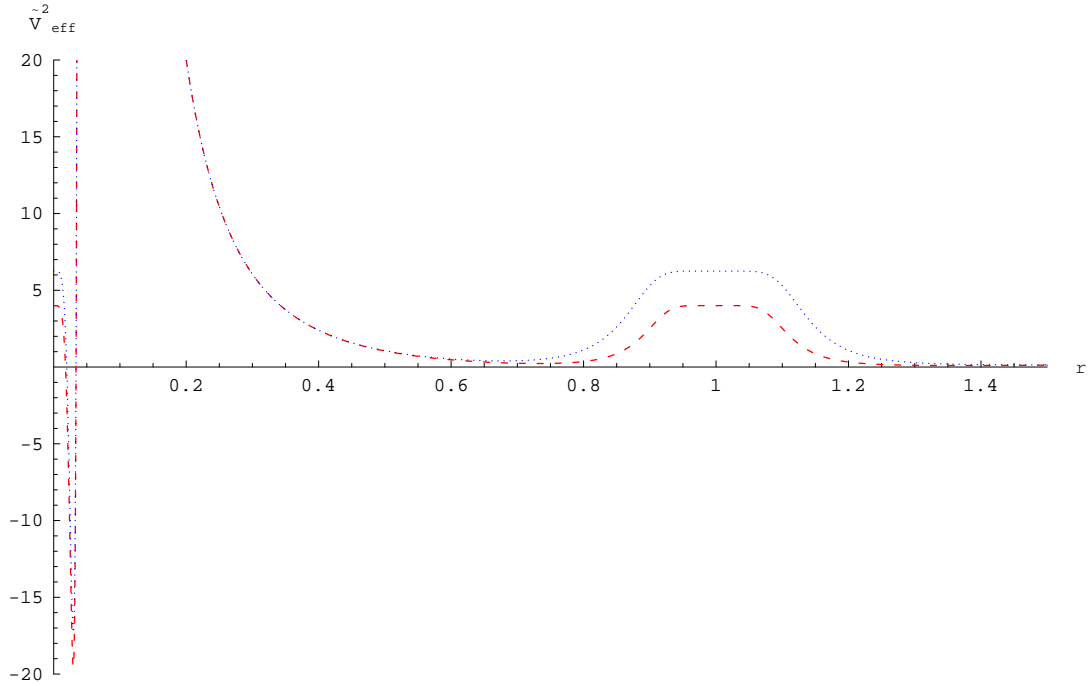


Figure 4: $V_{eff}^2(\rho)$ per unit of particle rest mass m for $\lambda = 0.1$, $\epsilon = 10^{-3}$, $\eta^2 = 1$ and the values $\tilde{E} = 2.5$ (dotted line) and $\tilde{E} = 2$ (dashed line).

from $\rho = 2$ to $\rho = \rho_A$, the possible presence of charge in the source essentially leaves the barrier at the external horizon unchanged. This means that all the remarks about the formation of black holes made in [26] with regard to the Schwarzschild metric can be extended to the “low charged” Reissner–Nordström space. Classically, the presence of the barrier forbids the formation of a black hole by the accretion of matter, unless the latter is transformed first into massless particles and these are absorbed by the star at a rate higher than the corresponding re-emission rate. A recent calculation [28] shows that even the quantum properties of matter do not alter this behaviour and that gravitational collapse would at least be slowed down. Nonetheless the shell would still retain the appearances of a very intensely radiating, charged, compact object ($\rho_A \leq 2$).

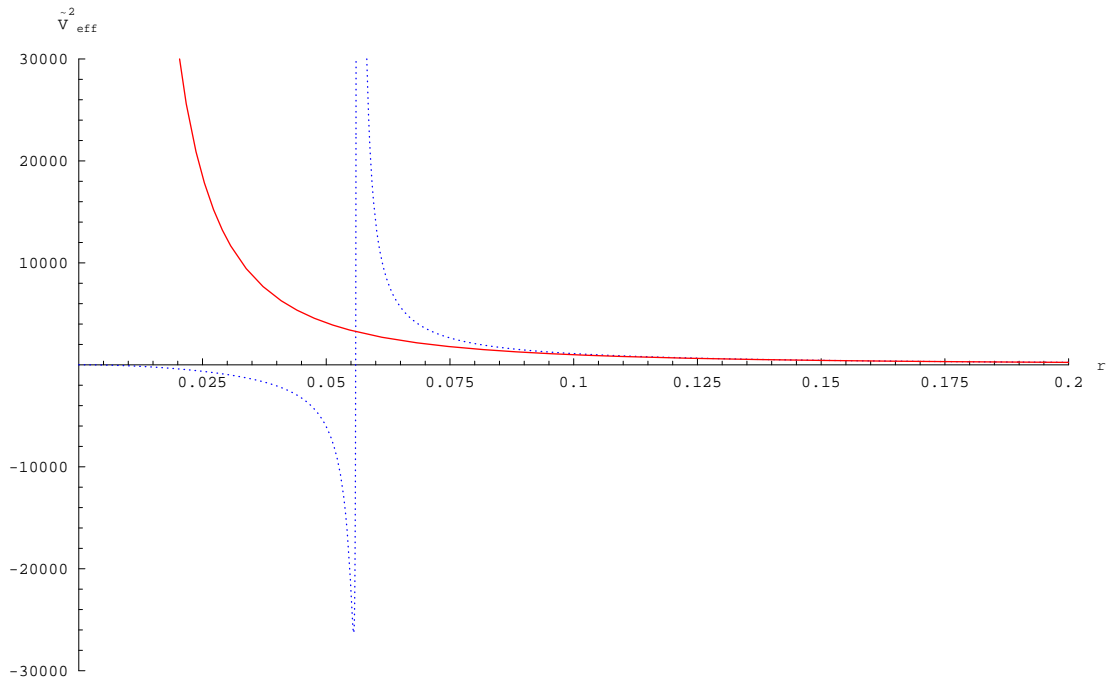


Figure 5: $V_{eff}^2(\rho)$ per unit of particle rest mass m for $\lambda = 10^{-2}$, $\epsilon = 10^{-3}$, $\eta^2 = 10$ and $\tilde{E} = 2.5$ (dotted line). The solid line represents the Reissner–Nordström potential for the same values of the parameters.

Besides confirming what found in [26] about black hole dynamics, the application of Caianiello model to the Reissner–Nordström metric offers other interesting aspects.

Since the shell is impervious to charged matter of both signs, the charge of the source can not change. It is not possible, at present, to speculate on the behaviour of electron-positron pairs produced in the immediate vicinity of a source with sufficiently high charge.

A barrier on a horizon is always accompanied by a singularity on the side where g_{00} is negative. These divergences are even present in the curvature invariants, so they must be considered as physical singularities of the effective metric. They are however inaccessible to massive probes and only regard the

behaviour of matter inside a black hole, if the latter somehow formed. It is however doubtful that the Reissner–Nordström solution would adequately describe this type of matter.

For high values of the angular momentum the divergence in the effective potential becomes positive causing the formation of an infinite, repulsive potential barrier.

It is important to remember that the Reissner–Nordström space–time allows causality violations in cases c) (and d)) and in the region $\rho < \rho_B$ of a) and b)[29]. The impenetrability of the shell renders inaccessible those regions with physically inadmissible properties that violate causality.

Finally, MA alters the radius of the naked singularity, but not its nature and does not therefore embody any form of cosmic censorship.

Acknowledgments

Research supported by Ministero dell’Università e della Ricerca Scientifica of Italy, funds ex 40% and 60% DPR 382/80 and the Natural Sciences and Engineering Research Council of Canada.

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